Student name: DOAN NHAT QUANG

ID: 20235377

**Estimation Theory: Home work 5**

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Description automatically generated

1. **Find and**

Matlab code:

% Define system matrices

A = [1 0; 0 1]; % State transition matrix

d = [5; 5]; % Constant displacement

Q = [1 0.5; 0.5 1]; % Process noise covariance

H = [1 0]; % Measurement matrix

R = 1; % Measurement noise covariance

% Initial estimates (MATLAB indices start at 1, not 0)

xhat(:,1) = [0; 0]; % Initial state estimate

P(:,:,1) = eye(2); % Initial error covariance matrix

% Measurement sequence

z = [0 4 9 16];

% Kalman filter loop for 3 measurements

for i = 2:4

%%Noises

w=sqrtm(Q)\*randn(2,1);

v=sqrtm(R)\*randn(1);

% Prediction step

xhat\_minus(:,i) = A \* xhat(:,i-1) + d + w; % Predicted state

P\_minus(:,:,i) = A \* P(:,:,i-1) \* A' + Q; % Predicted error covariance

% Kalman gain

K(:,:,i) = P\_minus(:,:,i) \* H' / (H \* P\_minus(:,:,i) \* H' + R); % Kalman gain

% Update step

xhat(:,i) = xhat\_minus(:,i) + K(i) \* (z(i) - H \* xhat\_minus(:,i) - v); % Updated state estimate

P(:,:,i) = (eye(2) - K(:,:,i) \* H) \* P\_minus(:,:,i); % Updated error covariance

end

% Display the final state estimate and error covariance

x3hat=xhat(:,4) % Estimated state after 3rd measurement (x3)

P3=P(:,:,4) % Error covariance after 3rd measurement (P3)

* Results:

x3hat =

15.2791

14.8367

P3 =

0.6190 0.2857

* 1. 3.5357

1. **Draw 90% ellipse using**

For a 90% ellipse level, K=4.605.



Matlab code:

% Estimated state (mean)

m = xhat(:,4); % x3hat from the Kalman filter result

% Covariance matrix at step 3

C = P(:,:,4); % P3 from the Kalman filter result

% Constant for 90% ellipse

K = 4.605;

% Call the function to draw the ellipse

r = draw\_ellipse(C, m, K);

% Plot the ellipse

figure;

plot(r(1,:), r(2,:), 'b-', 'LineWidth', 2);

hold on;

% Plot the estimated state as a point

plot(m(1), m(2), 'rx', 'MarkerSize', 10, 'LineWidth', 2);

% Labels and title

xlabel('x\_1');

ylabel('x\_2');

grid on;

axis equal;